Area spectrum of Schwarzschild black hole inspired by noncommutative geometry

Shao-Wen Wei, Yu-Xiao Liu,*, Zhen-Hua Zhao, Chun-E Fu

Institute of Theoretical Physics, Lanzhou University, Lanzhou 730000, P. R. China
E-mail: weishaow06@lzu.cn liuyx@lzu.edu.cn zhaozhh09@lzu.cn fuche08@lzu.cn

Abstract: It is known that, in the noncommutative Schwarzschild black hole spacetime, the point-like object is replaced by the smeared object, whose mass density is described by a Gaussian distribution of minimal width $\sqrt{\theta}$ with θ the noncommutative parameter. The elimination of the point-like structures makes it quite different from the conventional Schwarzschild black hole. In this paper, we mainly investigate the area spectrum and entropy spectrum for the noncommutative Schwarzschild black hole with $0 \leq \theta \leq \left(\frac{M}{1.90412}\right)^2$. By the use of the new physical interpretation of the quasinormal modes of black holes presented by Maggiore, we obtain the quantized area spectrum and entropy spectrum with the modified Hod's and Kunstatter's methods, respectively. The results show that: (1) The area spectrum and entropy spectrum are discrete. (2) The spectrum spacings are dependent on the parameter $\frac{M}{\sqrt{\theta}}$. (3) The spacing of the area spectrum of the noncommutative Schwarzschild black hole is smaller than that of the conventional one. So does the spacing of the entropy spectrum. (4) The spectra from the two methods are consistent with each other.

Keywords: Black hole, thermodynamics, area spectrum.

*Corresponding author.

Contents

1. Introduction 1

2. Review of the noncommutative Schwarzschild black hole solution 2

3. Area and entropy spectra of noncommutative Schwarzschild black hole 5

4. Summary 9

1. Introduction

Recently, motivated by string theory arguments [1], noncommutative geometry has been studied extensively. An important application of the noncommutative geometry is the black hole spacetime. In the noncommutative black hole spacetime, the point-like structure is eliminated [2] and the point-like object is replaced by the smeared object.

The first noncommutative black hole solution, known as the noncommutative Schwarzschild black hole, was presented by Nicolini, Smailagic and Spallucci [3] five years ago. It was found that there exists no singularity of scalar curvature R at the origin, which is different from the conventional Schwarzschild black hole for the temperature diverges and the scalar curvature becomes arbitrarily large [4]. Further, one can show that there exist no scalar singularities in this noncommutative black hole background according to the classification of the singularities given by Cai and Wang [5]. Thermodynamic properties of the noncommutative black hole were studied in [6, 7, 8, 9, 10, 11, 12, 13]. All these results showed interesting features for the noncommutative black hole at the small radius, while they coincide with the conventional black hole at the large radius. The evaporation of the noncommutative black hole was studied in [14], where it was shown that the final remnant of an extremal black hole is a thermodynamically stable object, which is different from that of the conventional Schwarzschild black hole for no remnant exists at the end of the evaporation. It was also observed [7] that, in the regime $\frac{M}{\sqrt{\theta}} \gg 1$, the noncommutative entropy/area law will recover the standard Bekenstein-Hawking area law, i.e., $S = \frac{A}{4}$.

Another important difference between the noncommutative Schwarzschild black hole and the conventional one is that the noncommutative one may have two horizons, one degenerate horizon or no horizon for different values of parameters. So, in some sense, the noncommutative black hole behaves like those black holes with two horizons. The similarity of the thermodynamics between it and the Reissner-Nordström (RN) black hole was studied and the noncommutative parameter θ is identified as the charge of the black hole with a simple relation [9]. Other noncommutative black hole solutions were found and their thermodynamics were investigated in [15, 16].

Motivated by the earlier work, we would like to study the area and entropy spectra for the noncommutative Schwarzschild black hole and we want to known whether the noncommutative parameter θ has any effect one the area and entropy spectra. With the new physical interpretation of the quasinormal modes of black holes presented by Maggiore [17], we obtain the quantized area spectrum and entropy spectrum with the modified Hod's and Kunstatter's methods [18, 19], respectively. The results show that the noncommutative parameter θ indeed has impact on the area spectrum and entropy spectrum. In the small $\frac{M}{\sqrt{\theta}}$ limit, the spacings of the area and entropy spectra have significant difference between the nonmcommutative and conventional Schwarzschild black holes. And when $\frac{M}{\sqrt{\theta}} \gg 1$, the area and entropy spectra, respectively, are consistent with each other for the two black holes.

The paper is organized as follows. In Sec. 2, we introduce the noncommutative black hole and examine its thermodynamic quantities in detail. With the new interpretation of the quasinormal modes, we calculate the area spectrum and entropy spectrum for the noncommutative black hole in Sec. 3. Finally, the paper ends with a brief summary.

2. Review of the noncommutative Schwarzschild black hole solution

In this section, we would like to give a brief review on the noncommutative black hole. An important property of the noncommutative black hole spacetime is that it eliminates the point-like structures [2], and which is replaced by a smeared object and the effect of smearing is mathematically implemented by replacing the position Dirac-delta function with a Gaussian distribution of minimal width $\sqrt{\theta}$ everywhere. Inspired by this idea, the mass density of a static, spherically symmetric, smeared, particle-like gravitational source is thought in the form [3]

$$\rho_\theta = \frac{M}{(4\pi\theta)^{\frac{3}{2}}} \exp\left(-\frac{r^2}{4\theta}\right). \tag{2.1}$$

The total mass M is diffused throughout the region of linear size $\sqrt{\theta}$. Then the mass involved in a sphere with radius r is

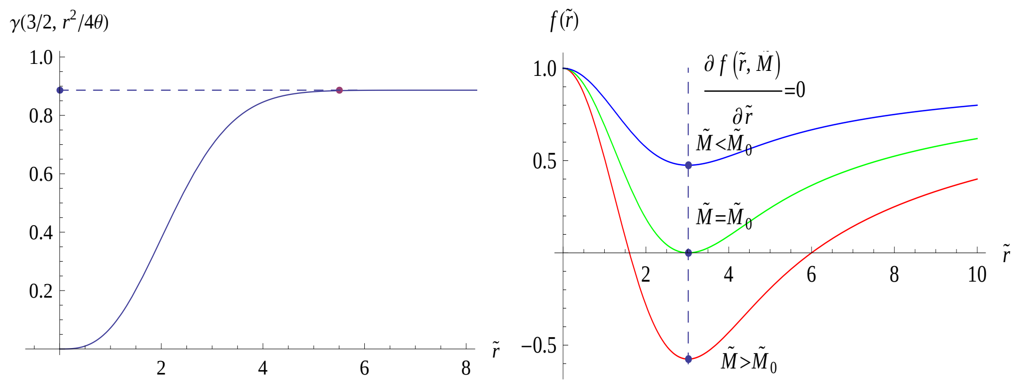
$$\begin{aligned} m(r) &= \int_0^r 4\pi r^2 \rho_\theta dr \\ &= \frac{2M}{\sqrt{\pi}} \gamma(\frac{3}{2}, \frac{r^2}{4\theta}), \end{aligned} \tag{2.2}$$

where $\gamma(\frac{3}{2}, \frac{r^2}{4\theta})$ is the lower incomplete Gamma function defined by

$$\gamma(\frac{3}{2}, \frac{r^2}{4\theta}) \equiv \int_0^{\frac{r^2}{4\theta}} t^{\frac{1}{2}} e^{-t} dt. \tag{2.3}$$

The energy-momentum tensor $T^\mu{}_\nu$ describing a static, spherically symmetric noncommutative black hole spacetime is [3]

$$T^\mu{}_\nu = \mathrm{diag}\bigg(-\rho_\theta, -\rho_\theta, -\rho_\theta - \frac{1}{2} r\partial_r \rho_\theta, -\rho_\theta - \frac{1}{2} r\partial_r \rho_\theta\bigg), \tag{2.4}$$



(a) (b)

Figure 1: (a) The lower incomplete Gamma function vs $\tilde{r} = \frac{r}{\sqrt{\theta}}$. When $\frac{r}{\sqrt{\theta}}$ is greater than 5.5, $\gamma(\frac{3}{2}, \frac{\tilde{r}^2}{4})$ will close to it's maximum $\frac{\sqrt{\pi}}{2}$. (b) The metric function $f(\tilde{r})$ vs $\tilde{r}$ with different values of the redefined mass $\tilde{M}$.

which is found to satisfy the conservation condition $T^{\mu\nu};\nu = 0$. The metric of the noncommutative Schwarzschild black hole is taken as

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2). \tag{2.5}$$

Solving the Einstein equations with this metric and the energy-momentum tensor (2.4), we could obtain the explicit form of the metric function $f(r)$, which is given by [3]

$$f(r) = 1 - \frac{2m(r)}{r} = 1 - \frac{4M}{r\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right). \tag{2.6}$$

Note that this black hole spacetime is closely dependent on the noncommutative parameter θ. However, there should be a natural restriction that the metric (2.5) should recover the conventional Schwarzschild black hole as $\theta \to 0$. For such purpose, we plot the lower incomplete Gamma function in Fig. 1(a). We could see that when $r/\sqrt{\theta}$ is greater than 5.5, the value of Gamma function will be closer to $\frac{\sqrt{\pi}}{2}$. Thus the metric function becomes $f(r) \approx 1 - \frac{2M}{r}$ for small θ, which is just the conventional Schwarzschild case.

From (2.6), it is obvious that the metric function contains the noncommutative parameter θ, however we can hide it by introducing the redefined mass and radial coordinate:

$$\tilde{M} = \frac{M}{\sqrt{\theta}}, \quad \tilde{r} = \frac{r}{\sqrt{\theta}}. \tag{2.7}$$

Then, the metric function becomes

$$f(\tilde{r}) = 1 - \frac{4\tilde{M}}{\tilde{r}\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{\tilde{r}^2}{4}\right), \tag{2.8}$$

where the noncommutative parameter θ has been successfully hidden. The behavior of $f(\tilde{r})$ is plotted against the radial coordinate $\tilde{r}$ with different values of $\tilde{M}$ in Fig. 1(b). The

horizons are determined by $f(\tilde{r}) = 0$. So, the horizons are located at the zero points of $f(\tilde{r}) = 0$, which are shown in Fig. 1(b). Form the figure, we could explicitly see that there are two horizons for $\tilde{M} > \tilde{M}_0$, while one degenerate horizon for $\tilde{M} = \tilde{M}_0$ and no horizon for $\tilde{M} < \tilde{M}_0$. Thus, $\tilde{M}_0$ can be regarded as an extremal mass for it splits the whole region into the non-extremal black hole and the naked singularity regions. Assume the black hole event horizon is existed, then it is given by

$$\tilde{r}_h = \frac{4\tilde{M}}{\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{\tilde{r}_h^2}{4}\right). \tag{2.9}$$

This is an iterative equation and has no analytical solution. However, one could obtain an approximate solution with an iteration method, i.e.,

$$r_h = 2M\left[1 - e^{-\frac{M^2}{\theta}}\left(\frac{2M}{\sqrt{\pi\theta}} + \mathcal{O}(\frac{\sqrt{\theta}}{M})\right)\right]. \tag{2.10}$$

Now, we would like to determine the values of the extremal mass $\tilde{M}_0$ and the corresponding radius $\tilde{r}_0$. From Fig. 1(b), we could see that there exists one minimum value of $f(\tilde{r})$ for each mass $\tilde{M}$. When $\tilde{M} > \tilde{M}_0$, the minimum is negative, while it is zero and is positive for $\tilde{M} = \tilde{M}_0$ and $\tilde{M} < \tilde{M}_0$, respectively. So, the extremal point must lie on the line determined by

$$\frac{\partial f(\tilde{r})}{\partial \tilde{r}} = 0. \tag{2.11}$$

Another condition to determine the extremal point is

$$f(\tilde{r}) = 0. \tag{2.12}$$

By solving these two equations (2.11) and (2.12), we can determine $\tilde{M}_0$ and $\tilde{r}_0$ uniquely. Substituting the metric function $f(\tilde{r})$ into (2.11), we derive

$$\tilde{r}^3 e^{-\frac{\tilde{r}^2}{4}} = 4\gamma\left(\frac{3}{2}, \frac{\tilde{r}^2}{4}\right). \tag{2.13}$$

Interestingly, this equation does not contain the mass parameter $\tilde{M}$. So, the line determined by (2.11) is essentially a vertical line in Fig. 1(b). It is not hard to understand that the root of Eq. (2.13) is just the value of $\tilde{r}_0$ and we obtain $\tilde{r}_0 = 3.02244$ numerically. Substituting $\tilde{r}_0$ into (2.12), we get the extremal mass $\tilde{M}_0 = 1.90412$. Or, the extremal point can also be expressed as $r_0 = 3.02244\sqrt{\theta}$ and $M_0 = 1.90412\sqrt{\theta}$. This result is exactly consistent with that of [9]. On the other hand, we can obtain the range θ for a black hole,

$$0 \leq \theta \leq \left(\frac{M}{1.90412}\right)^2. \tag{2.14}$$

Following, we would like to examine the thermodynamics quantities for this noncommutative black hole solution. The Hawking temperature T defined by $T = \frac{\partial_r f(r)}{4\pi}\mid_{r_h}$ is

read

$$
\begin{aligned}
T &= \frac{1}{4\pi r_h}\left(1-\frac{r_h^3}{4\theta^{3/2}\gamma(3/2,r_h^2/4\theta)}e^{-r_h^2/4\theta}\right)\\
&= \frac{1}{8\pi M}-\frac{M}{2\sqrt{\pi^3}\theta}e^{-M^2/\theta}\left(\frac{M}{\sqrt{\theta}}+\mathcal{O}(\frac{\sqrt{\theta}}{M})\right).
\end{aligned}
\tag{2.15}
$$

When $\frac{M}{\sqrt{\theta}} \gg 1$, it will return to the conventional Schwarzschild case and give $T_{\mathrm{Sch}}=\frac{1}{8\pi M}$. It was shown that the Benkenstein-Hawking entropy/area law is held for the noncommutative black hole [7]. Thus, the entropy is

$$
\begin{aligned}
S &= \frac{A}{4}=\pi r_h^2\\
&= 4\pi M^2-16\sqrt{\pi}M^2 e^{-\frac{M^2}{\theta}}\left(\frac{M}{\sqrt{\theta}}+\mathcal{O}(\frac{\sqrt{\theta}}{M})\right).
\end{aligned}
\tag{2.16}
$$

Heat capacity is a key quantity to measure the thermal stability of a black hole. Generally, a black hole with positive heat capacity can be stable existed in a heat bath, while a negative one will be all evaporated when a perturbation appears. Form (2.15), we can obtain the heat capacity:

$$
C_\theta=\left(\frac{dT}{dM}\right)^{-1}=-\frac{1}{8\pi M^2}+\frac{M^2-\theta}{\sqrt{\pi^3}\theta^2}e^{-\frac{M^2}{\theta}}\left(\frac{M}{\sqrt{\theta}}+\mathcal{O}(\frac{\sqrt{\theta}}{M})\right).
\tag{2.17}
$$

For the conventional Schwarzschild black hole, the heat capacity $C_{\mathrm{Sch}}=-\frac{1}{8\pi M^2}$. The negative heat capacity C_{Sch} implies an unstable black hole. However, for the noncommutative black hole, the capacity C_θ is positive for $M\in(1.90412\sqrt{\theta},2.3735\sqrt{\theta})$. So, the noncommutative black hole can stably exist in the range. It is also clear that, for $\frac{M}{\sqrt{\theta}}\gg 1$, the heat capacity C_θ will recover the conventional one.

Here, we have checked the thermodynamics quantities for the noncommutative black hole. It is shown that, different from the conventional one, the noncommutative black hole can stably exist in a heat bath in some range. It is also worth to point out that, for $\frac{M}{\sqrt{\theta}}\gg 1$, each thermodynamics quantity of the noncommutative black hole is identical to that of the conventional black hole.

3. Area and entropy spectra of noncommutative Schwarzschild black hole

The quantization of the black hole horizon area and entropy is an old but very interesting topic. Hod was one of the first to consider this problem ten years ago. He combined the perturbations of black holes with the principles of quantum mechanics and statistical physics in order to derive the quantum of the black hole area spectrum. With this idea, he obtained the area spectrum $A=4l_p^2\ln 3\cdot n$ [18].

On the other hand, Bekenstein first pointed out that the black hole horizon area is an adiabatic invariant [20] and the spacing of the area spectrum obtained from this viewpoint is $\Delta A=8\pi l_p^2$. Moreover, given a system with energy E and vibrational frequency $\omega(E)$,

the ratio $\frac{E}{\omega(E)}$ is a nature adiabatic invariant [19]. And using the Bohr-Sommerfeld quantization, Kunstatter [19] derived an equally spaced entropy spectrum for the D-dimensional Schwarzschild black hole. Subsequently, Hod's and Kunstatter's methods rejuvenated the interest on the study of the quantization of black hole area and entropy and the methods had been extended to other black holes [21, 22, 23, 24, 25, 26, 27, 29, 28, 31, 32].

Very recently, Maggiore presented a new physical interpretation for the quasinormal modes of black holes [17]. He suggested that the proper frequency of the equivalent harmonic oscillator, which is interpreted as the quasinormal mode frequency $\omega(E)$, should be of the form:

$$\omega(E) = \sqrt{|\omega_R|^2 + |\omega_I|^2}. \tag{3.1}$$

The form of the proper frequency for the quasinormal modes was first presented in [33]. Note that, for the case of long-lived quasinormal modes ($\omega_I \to 0$), we have $\omega(E) = |\omega_R|$, approximately. However, for the case of highly excited quasinormal modes ($|\omega_I| \gg |\omega_R|$), the natural selection should be $\omega(E) = |\omega_I|$. With this new physical interpretation of the quasinormal modes, the area spectrum of the Kerr black hole was obtained by Vagenas [22] with the modified Hod's and Kunstatter's methods, respectively. The spacing of the area spectrum calculated with the modified Hod's method is equally spaced, while it is non-equidistant and depends on the angular momentum parameter J employing the Kunstatter's method. The two methods give different spacings of the area spectrum. At the same time, it was Medved [23] who pointed out the difference of these results. He argued that the Kunstatter's method is only effective for the non-extremal Kerr black hole. The reason is that the quantum number n appearing in the Bohr-Sommerfeld quantization condition is actually a measure of the areal deviation from extremality for the black hole. Thus, the calculation of the Kunstatter's method is restricted to the case $M^2 \gg J$. In the spirit of this idea, the two methods give the same area spectrum, which is equally spaced and angular momentum parameter J-independent area spectrum.

Other charged or rotating black holes were also studied with the two methods [29]. Following the Kunstatter's method, the equally spaced area spectra were obtained for the non-extremal black holes. For the stringy charged Garfinkle-Horowitz-Strominger black hole, we showed that the area spectrum and entropy spectrum are both equally spaced and independent of the charge q [27]. For other non-Einstein gravity theories, the entropy spectra were found to be equally spaced, while the area spectra were non-equidistant (the detail can be found in [24, 26, 30, 32]).

Note that the works discussed above are all for conventional black holes. Now we would like to investigate the area spectrum and entropy spectrum for the noncommutative black hole. Different from the conventional black hole, we want to know that whether the noncommutative parameter θ has any effect on the area spectrum and entropy spectrum of the noncommutative black hole.

With these questions, we start our calculation. First, we will study the area spectrum and entropy spectrum for the noncommutative black hole by using the modified Hod's method. The asymptotic quasinormal frequency for the noncommutative black hole has

been obtained in [6]:

$$\omega = T\ln 3 + i2\pi T\left(k+\frac{1}{2}\right). \tag{3.2}$$

For the temperature is closely dependent on the noncommutative parameter θ, so the quasinormal frequency ω also depends on the parameter θ. The change in the parameters of the noncommutative black hole is determined by

$$\Delta M = \hbar\Delta\omega, \tag{3.3}$$

where $\Delta\omega$ can be obtained from Eq. (3.2). Considering the transition $k-1\to k$ for the noncommutative black hole, we obtain

$$\Delta\omega \approx |\omega_I|_k - |\omega_I|_{k-1} = 2\pi T, \quad (k\gg 1). \tag{3.4}$$

Generally, the change in the black hole mass will create a change in the black hole area:

$$\Delta A = 32\pi M\left[1+\frac{4M^2-6\theta}{\sqrt{\pi}\theta}e^{-\frac{M^2}{\theta}}\left(\frac{M}{\sqrt{\theta}}+\mathcal{O}(\frac{\sqrt{\theta}}{M})\right)\right]\Delta M. \tag{3.5}$$

Recalling the expression of the temperature in (2.15), Eq. (3.5) can be be rewritten as

$$\Delta A = \frac{4}{T}\Delta M + e^{-\frac{M^2}{\theta}}\mathcal{O}(\frac{\sqrt{\theta}}{M}). \tag{3.6}$$

Substituting (3.3) into (3.6), we obtain the spacing of the area spectrum

$$\Delta A = 8\pi\hbar + e^{-\frac{M^2}{\theta}}\mathcal{O}(\frac{\sqrt{\theta}}{M}). \tag{3.7}$$

Neglecting the high order of $\frac{\sqrt{\theta}}{M}$, we can obtain a θ- independent spacing of the area spectrum. The area spectrum for the noncommutative black hole can be assumed in the form:

$$A_n = 8\pi\hbar\cdot n + e^{-\frac{M^2}{\theta}}\mathcal{O}(\frac{\sqrt{\theta}}{M}). \tag{3.8}$$

Correspondingly, the quantized entropy spectrum is obtained through the entropy/area law:

$$S_n = 2\pi\hbar\cdot n + e^{-\frac{M^2}{\theta}}\mathcal{O}(\frac{\sqrt{\theta}}{M}). \tag{3.9}$$

A couple of comments are in order here. First, the area and entropy spectra are dependent on the parameter θ. Second, neglecting the high order of $\frac{\sqrt{\theta}}{M}$ (i.e., the case that far from the extremal black hole), we get equally spaced area and entropy spectra, which is in full agreement with that of the Schwarzschild black hole given by Maggiore [17].

Next, we will study the area spectrum and entropy spectrum for the noncommutative black hole by employing the Kunstatter's method. The adiabatic invariant I of this black hole is of the form

$$I = \int \frac{dM}{\Delta\omega(E)}. \tag{3.10}$$

Here, $\Delta\omega(E)$ still takes the form (3.4). Substituting the thermodynamic quantities into (3.10), we obtain

$$\begin{aligned} I &= \int \frac{dM}{2\pi T} \\ &\approx \int 4M\left(1+\frac{4M^3}{\sqrt{\pi\theta^3}}e^{-\frac{M^2}{\theta}}\right)dM \\ &= 2M^2 - \frac{8M^2}{\sqrt{\pi}}e^{-\frac{M^2}{\theta}}\left(\frac{M}{\sqrt{\theta}}+\mathcal{O}(\frac{\sqrt{\theta}}{M})\right) \\ &= \frac{A}{8\pi} + e^{-\frac{M^2}{\theta}}\mathcal{O}(\frac{\sqrt{\theta}}{M}). \end{aligned} \tag{3.11}$$

With the Bohr-Sommerfeld quantization condition $I \approx n\hbar \quad (n \gg 1)$ [1], we get the area spectrum

$$A_n = 8\pi\hbar \cdot n + e^{-\frac{M^2}{\theta}}\mathcal{O}(\frac{\sqrt{\theta}}{M}). \tag{3.12}$$

Recalling the entropy/area law, $S = \frac{A}{4}$, the quantized entropy spectrum is obtained

$$S_n = 2\pi\hbar \cdot n + e^{-\frac{M^2}{\theta}}\mathcal{O}(\frac{\sqrt{\theta}}{M}). \tag{3.13}$$

Obviously, the results are consistent with (3.8) and (3.9), which are obtained from the modified Hod's method.

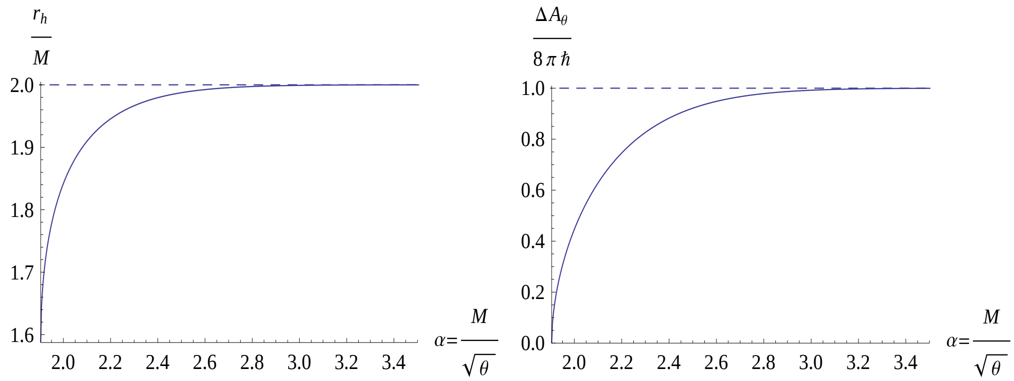
Now, we have obtained the quantized area spectrum and entropy spectrum for the noncommutative black hole with the modified Hod's and Kunstatter's methods for the case $\frac{M}{\sqrt{\theta}} \gg 1$. Neglecting the high order of $\frac{\sqrt{\theta}}{M}$, both the methods give the same equally spaced area spectrum and entropy spectrum, which are in full agreement with that of the Schwarzschild black hole given by Maggiore [17].

Note that our results above are all for $\frac{M}{\sqrt{\theta}} \gg 1$. Here, we would like to give a brief discussion with the numerical method for small $\frac{M}{\sqrt{\theta}}$ limit. Note the range of θ (2.14), we define a new parameter $\alpha = \frac{M}{\sqrt{\theta}} \geq 1.90412$. The behavior of r_h can be found in Fig. 2(a). From it, we can see that, the horizon $r_h < r_{\text{Sch}} = 2M$ and there exists a minimum horizon at $r_h = 1.58732M$, which corresponds to the extremal black hole. For the case $\frac{M}{\sqrt{\theta}} \gg 1$, the Schwarzschild black hole horizon will be recovered. On the other hand, with some calculations, we can express the area and entropy spectra in the forms

$$A_n = \Delta A_\theta \cdot n, \tag{3.14}$$

$$S_n = \Delta S_\theta \cdot n. \tag{3.15}$$

[1] The quantum number n is actually a measure of the areal deviation from extremality for the black hole. So, the black hole here is required to be far from its extremal case, i.e., $M \gg M_0$.



(a) (b)

Figure 2: (a) $\frac{r_h}{M}$ vs $\alpha = \frac{M}{\sqrt{\theta}}$ and (b) the area spacing vs $\alpha = \frac{M}{\sqrt{\theta}}$. The extremal black hole locates at $\alpha = 1.90412$ and the minimum horizon $r_h = 1.58732M$.

We can see that the area and entropy spectra are discrete, however their spacings are dependent on the parameter θ. The spacing of the area spectrum ΔA_θ is described in Fig. 2(b). It is monotonically increasing with $\frac{M}{\sqrt{\theta}}$. In the large $\frac{M}{\sqrt{\theta}}$ limit, the spacing of area spectrum $\Delta A_\theta = 8\pi\hbar$, which is consistent with (3.8). While for the small $\frac{M}{\sqrt{\theta}}$, the area spectrum closely depends on $\frac{M}{\sqrt{\theta}}$. For $S = \frac{A}{4}$, the spacing of the entropy spectrum has the similar behavior as the spacing of the area spectrum.

4. Summary

In this paper, we mainly deal with the noncommutative Schwarzschild black hole spacetime, where the point-like structure is eliminated and the point-like object is replaced by a smeared object. This special property makes the noncommutative Schwarzschild black hole behaves very different from the conventional one. We first examine the thermodynamics quantities of the noncommutative black hole. Its behaviors are very different from the conventional one near the extremal case, while they meet each other far from the extremal case. The quantization of the area and entropy for the noncommutative black hole is also studied. We calculate the area spectrum and entropy spectrum with the modified Hod's and the Kunstatter's methods, respectively. The results show that (1) The area spectrum and entropy spectrum are discrete. (2) The spectrum spacings are dependent on the parameter $\frac{M}{\sqrt{\theta}}$. (3) The spacing of the area spectrum of the noncommutative Schwarzschild black hole is smaller than that of the conventional one. So does the spacing of the entropy spectrum. (4) The spectra from the two methods are the same. Especially, when $\frac{M}{\sqrt{\theta}} \gg 1$, the area and entropy spectra are consistent with that of the conventional Schwarzschild black hole. These results can help us to further understand the properties of the noncommutative black hole spacetime.

Acknowledgments

This work was supported by the Program for New Century Excellent Talents in University, the National Natural Science Foundation of China (No. 11075065), the Huo Ying-Dong Education Foundation of Chinese Ministry of Education (No. 121106), the Doctoral Program Foundation of Institutions of Higher Education of China (No. 20090211110028), and the Fundamental Research Funds for the Central Universities (No. lzujbky-2009-54 and No. lzujbky-2009- 163).

References

[1] H. S. Snyder, *Quantized Space-Time*, Phys. Rev. **71** (1947) 38.

[2] A. Smailagic and E. Spallucci, *Feynman Path Integral on the Noncommutative Plane*, J. Phys. **A 36** (2003) L467; A. Smailagic and E. Spallucci, *UV divergence-free QFT on noncommutative plane*, J. Phys. **A 36** (2003) L517.

[3] P. Nicolini, A. Smailagic and E. Spallucci, *The fate of radiating black holes in noncommutative geometry*, ESA Spec. Publ. **637** (2006) 11.1; P. Nicolini, A. Smailagic and E. Spallucci, *Noncommutative geometry inspired Schwarzschild black hole*, Phys. Lett. **B 632** (2006) 547.

[4] P. Nicolini, *A model of radiating black hole in noncommutative geometry*, J. Phys. **A 38** (2005) L631; A. Gruppuso, *Newton's law in an effective non commutative space-time*, J. Phys. **A 38** (2005) 2039.

[5] R. G. Cai and A. Wang, *Singularities in Horava-Lifshitz theory*, Phys. Lett. **B 686** (2010) 166.

[6] P. R. Giri, *Asymptotic quasinormal modes of a noncommutative geometry inspired Schwarzschild black hole*, Int. J. Mod. Phys. **A 22** (2007) 2047; K. Nozari and B. Fazlpour, *Thermodynamics of an Evaporating Schwarzschild Black Hole in Noncommutative Space*, Mod. Phys. Lett. **A 22** (2007) 2917; E. Harikumar and Victor O. Rivelles, *Noncommutative Gravity*, Class. Quant. Grav. **23** (2006) 7551; S. Kar, *Noncommutative brane-world, (Anti) de Sitter vacua and extra dimensions*, JHEP **0610** (2006) 052; E. D. Grezia, G. Esposito and G. Miele, *The scalar wave equation in a non-commutative spherically symmetric space-time*, Int. J. Geom. Meth. Mod. Phys. **5** (2008) 33; K. Nozari and S. Akhshabi, *Orbits of particles in noncommutative Schwarzschild spacetime*, Europhys. Lett. **80** (2007) 20002.

[7] R. Banerjee, B. R. Majhi and S. Samanta, *Noncommutative Black Hole Thermodynamics*, Phys. Rev. **D 77** (2008) 124035; R. Banerjee, B. R. Majhi and S. K. Modak, *Noncommutative Schwarzschild Black Hole and Area Law*, Class. Quant. Grav. **26** (2009) 085010; R. Banerjee, B. Chakraborty, S. Ghosh, P. Mukherjee and S. Samanta, *Topics in Noncommutative Geometry Inspired Physics*, Found. Phys. **39** (2009) 1297; R. Banerjee, S. Gangopadhyay and S. K. Modak, *Voros product, Noncommutative Schwarzschild Black Hole and Corrected Area Law*, Phys. Lett. **B 686** (2010) 181.

[8] K. Nozari and S. H. Mehdipour, *Hawking Radiation as Quantum Tunneling from Noncommutative Schwarzschild Black Hole*, Class. Quant. Grav. **25** (2008) 175015.

[9] W. Kim, Edwin J. Son and M. Yoon, *Thermodynamic similarity between the noncommutative Schwarzschild black hole and the Reissner-Nordstrom black hole*, JHEP **0804** (2008) 042.

[10] B. Vakili, N. Khosravi and H. R. Sepangi, *Thermodynamics of noncommutative de Sitter spacetime*, Int. J. Mod. Phys. **D 18** (2009) 159.

[11] M. Buric and J. Madore, *Spherically Symmetric Noncommutative Space: $d = 4$*, Eur. Phys. J. **C 58** (2008) 347.

[12] W. H. Huang and K. W. Huang, *Thermodynamics on Noncommutative Geometry in Coherent State Formalism*, Phys. Lett. **B 670** (2009) 416.

[13] M. Park, *Smeared Hairs and Black Holes in Three-Dimensional de Sitter Spacetime*, Phys. Rev. **D 80** (2009) 084026;

K. Nozari and S. H. Mehdipour, *Parikh-Wilczek Tunneling from Noncommutative Higher Dimensional Black Holes*, JHEP **0903** (2009) 061; J. J. Oh and C. Park, *Noncommutative Black Holes from Gravitationally Collapsing Shells*, JHEP **1003** (2010) 086; I. Arraut, D. Batic and M. Nowakowski, *Maximal extension of the Schwarzschild spacetime inspired by noncommutative geometry*, arXiv:1001.2226[gr-qc]; S.H. Mehdipour, *Parikh-Wilczek tunneling as massive particles from noncommutative Schwarzschild black hole*, Commun. Theor. Phys. **52** (2009) 865; D. M. Gingrich, *Noncommutative geometry inspired black holes in higher dimensions at the LHC*, JHEP **1105** (2010) 022.

[14] H. Garcia-Compean and C. Soto-Campos, *Noncommutative Effects in the Black Hole Evaporation in Two Dimensions*, Phys. Rev. **D 74** (2006) 104028; E. D. Grezia, G. Esposito and G. Miele, *Gravitational amplitudes in black-hole evaporation: the effect of non-commutative geometry*, Class. Quant. Grav. **23** (2006) 6425; E. D. Grezia, G. Esposito and G. Miele, *Black hole evaporation in a spherically symmetric non-commutative space-time*, J. Phys. **A 41** (2008) 164063; Y. S. Myung, Y. W. Kim and Y. J. Park, *Thermodynamics and evaporation of the noncommutative black hole*, JHEP **0702** (2007) 012; R. Casadio and P. Nicolini, *The decay-time of non-commutative micro-black holes*, JHEP **0811** (2008) 072.

[15] T. G. Rizzo, *Noncommutative Inspired Black Holes in Extra Dimensions*, JHEP **0609** (2006) 021; S. Kar and S. Majumdar, *Noncommutative D_3-brane, Black Holes and Attractor Mechanism*, Phys. Rev. **D 74** (2006) 066003; K. Nozari and B. Fazlpour, *Reissner-Nordstrom Black Hole Thermodynamics in Noncommutative Spaces*, ActaPhys. Polon. **B 39** (2008) 1363; K. Nozari and S. H. Mehdipour, *Noncommutative Inspired Reissner-Nordstrom Black Holes in Large Extra Dimensions*, Commun. Theor. Phys. **53** (2010) 503; S. Ansoldi, P. Nicolini, A. Smailagic and E. Spallucci, *Noncommutative geometry inspired charged black holes*, Phys. Lett. **B 645** (2007) 261; E. Spallucci, A. Smailagic and P. Nicolini, *Non-commutative geometry inspired higher-dimensional charged, black holes*, Phys. Lett. **B 670** (2009) 449; P. Nicolini, *Noncommutative Black Holes, The Final Appeal To Quantum G ravity: A Review*, Int. J. Mod. Phys. **A 24** (2009) 1229; P. Nicolini and E. Spallucci, *Noncommutative geometry inspired dirty black holes*, Class. Quant. Grav. **27** (2010) 015010.

[16] E. D. Grezia and G. Esposito, *Non-commutative Kerr black hole*, arXiv:0906.2303[hep-th]; S. A. Alavi, *Reissner-Nordstrom black hole in noncommutative spaces*, Acta Phys. Polon. **B 40** (2009) 2679; T. Asakawa and S. Kobayashi, *Noncommutative Solitons of Gravity*, Class. Quantum Grav. **27** (2010) 105014. J. M. Tejeiro and A. Larranaga, *Noncommutative Geometry Inspired Rotating Black Hole in Three Dimensions*, arXiv:1004.1120[gr-qc]; A. Larranaga and J. M. Tejeiro, *Three Dimensional Charged Black Hole Inspired by Noncommutative Geometry*, arXiv:1004.1608[gr-qc]; S. H. Mehdipour, *Charged Particles' Tunneling from Noncommutative Charged Black Hole*, Int. J. Mod. Phys. **A 25** (2010) 5543; A. Smailagica and E. Spallucci, *"Kerrr" black hole: The lord of the string*, Phys. Lett. **B 688** (2010) 82.

[17] M. Maggiore, *Physical Interpretation of the Spectrum of Black Hole Quasinormal Modes*, Phys. Rev. Lett. **100** (2008) 141301.

[18] S. Hod, *Bohr's Correspondence Principle and the Area Spectrum of Quantum Black Holes*, Phys. Rev. Lett. **81** (1998) 4293; S. Hod, *Best approximation to a reversible process in black-hole physics and the area spectrum of spherical black holes*, Phys. Rev. **D 59** (1998) 024014.

[19] G. Kunstatter, *d-Dimensional Black Hole Entropy Spectrum from Quasi-Normal Modes*, Phys. Rev. Lett. **90** (2003) 161301.

[20] J. D. Bekenstein, *The Quantum Mass Spectrum of the Kerr Black Hole*, Lett. Nuovo Cimento **11** (1974) 467; J. D. Bekenstein and V. Mukhanov, *Spectroscopy of the quantum black hole*, Phys. Lett. **B 360** (1995) 7; J. D. Bekenstein, *Quantum Black Holes as Atoms*, arXiv:gr-qc/9710076; J. D. Bekenstein, *Black Holes: Classical Properties, Thermodynamics and Heuristic Quantization*, arXiv:gr-qc/9808028.

[21] O. Dreyer, *Quasinormal Modes, the Area Spectrum, and Black Hole Entropy*, Phys. Rev. Lett. **90** (2003) 081301; A. P. Polychronakos, *Area spectrum and quasinormal modes of black holes*, Phys. Rev. **D 69** (2004) 044010; M. R. Setare, *Non-Rotating BTZ Black Hole Area Spectrum from Quasi-normal Modes*, Class. Quant. Grav. **21** (2004) 1453; M. R. Setare, *Area Spectrum of Extremal Reissner-Nordström Black Holes from Quasi-normal Modes*, Phys. Rev. **D 69** (2004) 044016; M. R. Setare, *Near Extremal Schwarzschild-de Sitter Black Hole Area Spectrum from Quasi-normal Modes*, Gen. Rel. Grav. **37** (2005) 1411; M. R. Setare and E. C. Vagenas, *Area Spectrum of Kerr and extremal Kerr Black Holes from Quasinormal Modes*, Mod. Phys. Lett. **A 20** (2005) 1923; M. R. Setare, *Area Spectrum of Near Extremal Black Branes from Quasi-normal Modes* , Int. J. Theor. Phys. **44** (2005) 1365; S. Lepe and J. Saavedra, *Quasinormal modes, superradiance and area spectrum for 2+1 acoustic black holes*, Phys. Lett. **B 617** (2005) 174.

[22] E. C. Vagenas, *Area spectrum of rotating black holes via the new interpretation of quasinormal modes*, JHEP **0811** (2008) 073.

[23] A. J. M. Medved, *On the Kerr quantum area spectrum*, Class. Quant. Grav. **25** (2008) 205014.

[24] D. Kothawala, T. Padmanabhan and S. Sarkar, *Is gravitational entropy quantized?*, Phys. Rev. **D 78** (2008) 104018.

[25] E. Abdalla, K. H. C. Castello-Branco and A. Lima-Santos, *Area Quantization in Quasi-Extreme Black Holes*, Mod. Phys. Lett. **A 18** (2003) 1435; T. Tamaki and H. Nomura, *The universal area spectrum in single-horizon black holes*, Phys. Rev. **D 70** (2004) 044041; T. Tanaka and T. Tamaki, *Black hole entropy for the general area spectrum*, arXiv:0808.4056[hep-th]; S. Fernando, *Spinning Dilaton Black Holes in 2 +1 Dimensions: Quasi-normal Modes and the Area Spectrum*, Phys. Rev. **D 79** (2009) 124026.

[26] S. W. Wei, R. Li, Y. X. Liu and J. R. Ren, *Quantization of Black Hole Entropy from Quasinormal Modes*, JHEP **0903** (2009) 076; S. W. Wei and Y. X. Liu, *Area Spectrum of the Large AdS Black Hole from Quasinormal Modes*, arXiv:0906.0908[hep-th].

[27] S. W. Wei, Y. X. Liu, K. Yang and Y. Zhong, *Entropy/Area spectra of the charged black hole from quasinormal modes*, Phys. Rev. **D 81** (2010) 104042.

[28] Y. Kwon and S. Nam, *Area spectra of the rotating BTZ black hole from quasinormal modes*, Class. Quant. Grav. **27** (2010) 125007; Y. Kwon and S. Nam, *Area spectra versus entropy spectra in black holes in topologically massive gravity*, Class. Quant. Grav. **27** (2010) 165011;

P. Gonzalez, E. Papantonopoulos and J. Saavedra, *Quasi-normal modes of scalar perturbations, mass and area spectrum of Chern-Simons black holes*, JHEP **1008** (2010) 050;

[29] Y. S. Myung, *Area spectrum of slowly rotating black holes*, Phys. Lett. **B 689** (2010) 42; A. Lopez-Ortega, *Area spectrum of the D-dimensional de Sitter spacetime*, Phys. Lett. **B 682** (2009) 85; A. Lopez-Ortega, *Area spectrum of the d-dimensional Reissner-Nordstrom black hole in the small charge limit*, arXiv:1003.4248[gr-qc].

[30] R. Banerjee, B. R. Majhi and E. C. Vagenas, *Quantum tunneling and black hole spectroscopy*, Phys. Lett. **B 686** (2010) 279; B. R. Majhi, *Hawking radiation and black hole spectroscopy in Horava-Lifshitz gravity*, Phys. Lett. **B 686** (2010) 49.

[31] W. Li, L. Xu and J. Lu, *Area spectrum of near-extremal SdS black holes via the new interpretation of quasinormal modes*, Phys. Lett. **B 676** (2009) 177; J. L. Jing and C. K. Ding, *Quasinormal spectrum and quantization of the Kerr-Newman black hole*, Chin. Phys. Lett. **25** (2008) 858; J. Wang and J. L. Jing, *Quantum Spectrum of Stationary Axisymmetric EinsteinCMaxwell DilatonCAxion Black Hole*, Chin. Phys. Lett. **22** (2005) 2766;

[32] M. R. Setare and D. Momeni, *Spacing of the entropy spectrum for KS Black hole in Horava-Lifshitz gravity*, arXiv:1002.0185[hep-th].

[33] B. Wang, C. Y. Lin and C. Molina, *Quasinormal behavior of massless scalar field perturbation in Reissner-Nordstrom Anti-de Sitter spacetimes*, Phys. Rev. **D 70** (2004) 064025; B. Wang, C. Molina and E. Abdalla, *Evolving of a massless scalar field in Reissner–Nordstrom Anti de Sitter spacetimes*, Phys. Rev. **D 63** (2001) 084001.